Optimal Design of Three-Link Planar Manipulators using Grashof's Criterion

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Chapter 3
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ABSTRACT

The design of robotic manipulators is dictated by a set of pre-determined task descriptions and performance parameters. These performance parameters are often defined in terms of workspace dexterity, manipulability, and accuracy. Many serial manipulator applications require that the manipulator have full dexterity about a work piece or a pre-defined trajectory, that is, to approach the given point within the workspace with all possible orientations about that point. Grashof's criterion defines the mobility of four-link closed chain mechanisms in relation to its link lengths. A simple assumption can convert a three-link serial manipulator into a four-link closed chain so that its mobility can be studied using Grashof's criterion. With the help of Grashof's criterion, it is possible not only to predict and simulate the mobility of a manipulator during its design, but also to map and identify the fully-dexterous regions within its workspace. Mapping of the dexterous workspace is helpful in efficient task placement and path planning. Next, the authors propose a simple algorithm using Grashof's criterion for determining the optimal link lengths of a three-link manipulator, in order to achieve full dexterity at the desired regions of the workspace. Finally, the authors test the generated design by applying joint angle limitations.

NOMENCLATURE

D: Dexterity index of the manipulator at a point.

D_{Mean}: Mean dexterity index over a region or trajectory.

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N: Number of points along the trajectory.

d_x, d_y, d_z: Dexterity indices about the X, Y and Z axis.

α, β, γ: Yaw, pitch and roll angels of the end-effector.

a, b, c, d: Link lengths of the four-bar kinematic chain.

l_1, l_2, l_3: Link lengths of the three-link planar manipulator.
θ_1, θ_2, θ_3: Joint angles of the three-link planar manipulator.
d: Distance between a task-point and base of the manipulator
d_{min}: Minimum distance between a task-point and base of the manipulator.
d_{max}: Maximum distance between a task-point and base of the manipulator.

1. INTRODUCTION

The problem of designing an optimal manipulator configuration is very complex, as the equations governing the motion of the end-effector in the workspace are both non-linear and complex, often having no closed solutions. Prototyping methods such as kinematic synthesis and numerical optimization are complex and very time consuming. The inherent complexity of kinematic synthesis has helped to make a strong case for rapid prototyping methods in which manipulators are designed with very specific performance requirements or tasks point specifications. Rapid prototyping allows designers to spend more time on design, simulation and evaluation of different manipulator configurations instead of solving mathematical models describing kinematics chains.

The study of mobility of closed chain mechanisms has interested researchers for a very long time. Understanding the mobility of chain mechanisms in relation to their link lengths can help us to design better and highly dexterous manipulators. In 1833, Grashof first introduced a simple rule to understand the mobility of four-link mechanisms [6]. This rule, commonly known as the Grashof's theorem, helps analyze the rotatability of links in a closed four-bar mechanism. This was further extended by Paul (1979), who introduced an inequality into the Grashof's theorem and proved that Grashof's criterion is both a necessary and sufficient condition for the existence of a crank in the four-bar mechanism (Chang, Lin, & Wu, 2005).

Researchers have applied Grashof's criterion to understand and study the workspace mobility of both closed and open chain planar mechanisms. Barker (1985), using Grasshof's criterion, classified four-bar planar mechanisms based on their mobility. Grashof's criterion was applied to the study of three-link planar mechanism by Li and Dai (2009). Furthermore, they developed equations for the orientation angle and presented a simple program to analyze the orientation angle for a manipulator, given the link parameters. The mobility and orientation of open chain mechanisms can also be analyzed using Grashof's criterion. Dai and Shah (2002, 2003) studied the mobility of serial manipulators by introducing a virtual ground link between the end-effector and the base so as to form a virtual closed chain. In (Li, & Dai, 2009; Dai, & Shah, 2003), the authors proposed workspace decomposition based on the orientation capability of the manipulator.

Grashof's Theorem has been extended to include more than four-bar chain mechanisms. Grashof's criterion for five bar chain was proposed by Ting (1986). Ting and Liu (1991) extended this work to evaluate the mobility of N-bar chain mechanisms. Nokleby and Podhorodeski (2001) applied Grashof's criterion for the optimized synthesis for five-bar mechanisms.

In this work we present a simple algorithm for the optimal design of a three-link planar manipulator, using Grashof's criterion. We begin by adding a virtual link to the three-link planar manipulator in order to make it a closed four-bar chain mechanism, so that Grashof's criterion can be applied. We evaluate the generated manipulator designs using dexterity index as a performance measure. Our proposed optimization algorithm generates the required link lengths such that the manipulator has maximum dexterity in the region specified by the user. This region of interest can either be a set of task points or a trajectory. Furthermore, we have also demonstrated, with the help of simulations, the influence of the serial chain link ordering on the dexterous workspace. Finally, we simulate our design under practical conditions such as joint angle limitations.

2. DEXTERITY INDEX AS A PERFORMANCE MEASURE

The performance evaluation of a robotic manipulator in its workspace is central to the design process of the manipulator. Both a bottom-up approach – designing a manipulator to meet a certain performance standards (Gosselin, 1992; Sobh, & Toundykov, 2004; Paden, & Sastry, 1988) and a top-down approach – optimal task placement in workspace of the manipulator (Santos, Steffen, & Saramago, 2010), have been thoroughly investigated. In either case, it is essential to define proper performance parameters and study the variation of these parameters in the workspace.

In this process, numerous parameters have been proposed to quantify and measure the performance of the manipulator in its workspace, such as dexterity index, manipulability index, condition number, minimum singular value, etc. (Tanev, & Stoyanov, 2000). Two parameters stand out in quantifying the manipulator's performance in its workspace: dexterity and manipulability. A manipulator has to be highly dexterous or manipulable in its workspace to meet high performance standards. Many applications of robotic manipulators require the manipulator to be fully dexterous to perform specified tasks.

The dexterity of a manipulator is defined as the number of ways in which any given point in the workspace can be approached by the end-effector. The dexterity index of a manipulator at a point in the workspace can be defined as a measure of a manipulator to achieve varying orientations at that point (Tanev, & Stoyanov, 2000).

In their work, Kumar and Waldron (1981) introduced the parameter dexterity index as another measure for manipulator performance. They defined dexterous workspace as *'the volume within which every point can be reached by the manipulator end-effector with any desired orientation'* (Tanev, & Stoyanov, 2000, p. 1). While the manipulability index is a function of both the manipulator configuration and joint angles (the manipulability index can be different at the same point if the manipulator has multiple inverse kinematic solutions), the dexterity index is only a function of the manipulator configuration.

The orientation at any given point in the workspace can be represented in terms of the yaw (α), pitch (β) and roll (γ) angles (Spong, & Vidyasagar, 1989) as:

$$R_{xyz} = R_{x,\gamma} R_{y,\beta} R_{z,\alpha} \tag{1}$$

All three of the angles have a range 0 - 2π to provide all possible orientations. The dexterity index can be defined as the summation of the dexterity indices about each of the axes (Tanev, & Stoyanov, 2000) given by Equation (2):

$$D = \frac{1}{3}(d_x + d_y + d_z) \tag{2}$$

$$D = \frac{1}{3}(\frac{\Delta\gamma}{2\pi} + \frac{\Delta\beta}{2\pi} + \frac{\Delta\alpha}{2\pi}) \tag{3}$$

where d_x, d_y, and d_z are X, Y and Z dexterity indices. Δα, Δβ and Δγ are the range of possible yaw, pitch and roll angles about a point.

As seen in Equation (3), the dexterity index can vary between a minimum of 0 to a maximum of 1, hence is a well bounded parameter. Therefore, points in the workspace with multiple inverse kinematic solutions will have a higher dexterity index when compared to points with unique solutions.

The mean dexterity index of a manipulator for a region of the workspace or trajectory can be defined as (Tanev, & Stoyanov, 2000):

$$DMean = \frac{\sum D}{N} \tag{4}$$

The manipulator is said to be completely dexterous at a given point if the dexterity at that point is equal to unity. Similarly, an area or point in the workspace can be said to be completely X-dexterous or Y-dexterous if d_x or d_y is equal to unity. In the case of a planar manipulator operating in the XY – plane

$$d_x = d_y = 0 \tag{5}$$

A simple algorithm for calculating the dexterity index of a workspace was proposed in (Tanev, & Stoyanov, 2000).

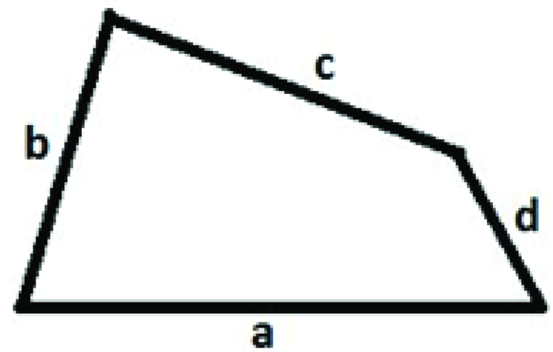
3. GRASHOF'S CRITERION

The study of mobility of a closed chain mechanism has interested researchers for a very long time. In 1833, Grashof first introduced a simple rule to understand the mobility of four-link mechanisms (Kumar, & Waldron, 1981). This rule, commonly known as Grashof's theorem, helps judge the rotatability of links in a four bar mechanism. This was further extended by Paul (1979), who introduced an inequality into Grashof's theorem and proved that Grashof's criterion is both a necessary and sufficient condition for the existence of a crank in the four-bar mechanism.

Consider a four-link kinematic chain consisting of four links a, b, c, and d, as shown in Figure 1. Let a be the longest link and d be the short link in the chain, such that $a>b\geq c>d$. According to Grashof's criterion, there exists at least one link that can fully revolve with respect to the other links if:

$$a+d\leq b+c \tag{6}$$

Figure 1. Four-link kinematic chain



i.e. the sum of the longest and the shortest link should be less than or equal to the sum of the other two links. And none of the links are fully revolute if:

$$a+d>b+c \tag{7}$$

Paul (1979) proved that this criterion was both a necessary and sufficient condition for the existence of a fully rotatable link in the chain. Such a mechanism is also known as a Grashof linkage (Chang, Lin, & Wu, 2005; Li, & Dai, 2009). In a Grashof linkage, the shortest link in the chain is the always fully revolvable with respect to the other links (Li, & Dai, 2009).

The complete classification and behavior of four-bar linkages is explained in (Chang, Lin, & Wu, 2005; Barker, 1985).

4. DESIGN OPTIMIZATION

It is impossible for a manipulator to be equally dexterous or highly dexterous at all points in the workspace. Some regions of the workspace have high dexterity while other regions can only be attained by a unique set of joint angles. Therefore, it is important that the manipulator be designed in such a way that it has maximum dexterity in its region of operation.

By design optimization we mean adjusting the link lengths in such a way that the manipulator has maximum dexterity in the region of interest. As the dexterity index is a function of the manipulator configuration, by optimizing the link lengths, the manipulator's dexterous workspace can be engineered. A three-link planar manipulator is a serial link chain with three revolute joints, as seen in Figure 2. The forward kinematic equations for a three-link planar manipulator are given by Equations (8), (9)

$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3) \quad (8)$$

$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3) \quad (9)$$

In order to apply Grashof's criterion, essentially meant for four-bar linkages, to design three-link planar manipulators, we assume the distance between the base of the manipulator and the center of the end-effector as the imaginary fourth link of the chain. The length of this imaginary fourth link is not constant and depends on the joint angles, which in turn determine the position of the end-effector in the workspace. Hence, the links lengths should be selected such that Grashof's criterion is satisfied at all points in the region of operation as d varies from d_{min} to d_{max}.

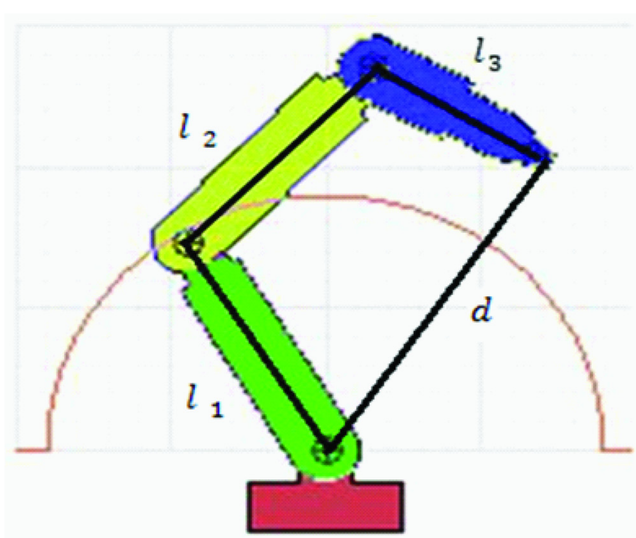
Consider a three-link planar manipulator with links l_1, l_2 and l_3, as shown in Figure 2. Let d be the variable radial distance between the base and the end-effector shown in Figure 2. In order to optimize the link lengths to achieve maximum dexterity in the area of interest or trajectory, we propose the following algorithm.

1. Let d_{max} be the maximum distance from the base and d_{min} minimum distance while following a trajectory such that $d_{min} \le d \le d_{max}$
2. Let l_3 be the shortest link in the manipulator such that $l_3 < d_{min}$. The length of l_3 is equal to the minimum link length practically possible this is determined by the other factors such as the size of the motors and loading on the manipulator.

Next, we calculate l_1 and l_2 as follows:

$$l_1 = \left(\frac{d_{\max} + l_3}{2}\right) + 1 \quad (10)$$

Figure 2. 3-Link planar manipulator



$$l_2 = \left(\frac{d_{\max} + l_3}{2} \right) \quad (11)$$

Since d_{max} and l_3 are finite and known l_1 and l_2 can easily be determined.

4. The dexterity index plot of the complete workspace is generated to check if the manipulator has maximum dexterity in the area of interest.

The link lengths are chosen such that the manipulator behaves as a Grashof's linkage as the radial distance from the base to the task point varies from d_{min} to d_{max}. l_1 and l_2 are chosen to be the larger links in the chain so that the last link or the end-effecter l3 is imparted full rotatability at all points within this range. The fourth link *d* is an imaginary link whose length depends on the position of the task point. Equations (10) and (11) give the minimal link lengths required to satisfy Grashof's criterion such that:

At $d=d_{max}$ we have $l_1>l_2>d>l_3$ and $l_1+l_3>l_2+d$

At $d=d_{min}$ we have $l_1>l_2>d>l_3$ and $l_1+l_3>l_2+d$

Section 5.2 discusses a condition with changed link order where the second link is the shortest link in the chain.

As long as Grashof's criterion is satisfied, the shortest link (end-effector) will be completely revolute, as seen in Figures 3 and 4. Figure 3 shows few of the infinite orientations that the manipulator can attain about a given point when Grashof's criterion is satisfied. The manipulator will obviously not behave as a Grashof's linkage at all points in the workspace since the fourth link in the chain *d* is not constant. Optimization ensures that the manipulator will behave as a Grashof's linkage in the region $d_{min} \leq d \leq d_{max}$, thereby making the end-effector completely revolute and hence, providing maximum dexterity to manipulator in the region.

As the task point keeps moving in the workspace, the radial distance between the base of the manipulator and the task point, the length of virtual fourth link 'd' keeps changing. Accordingly, depending of the position of the task point the shortest link in the chain will be as shown in Table 1.

It is important to mention here that Grashof's criterion only sets a condition for one of the links in the chain to be fully revolute with respect to the other links, but does not comment on the order of the links in the chain. Grashof's criterion helps only in deciding the link lengths, however the dexterity of the manipulator depends not only on the link lengths but also on their relative positioning in the kinematic chain.

5. EXAMPLE 1: TRAJECTORY

Let us consider a design problem where we need to design an optimal manipulator configuration while following a cubic trajectory joining the task points A (14, 14), B (12, 8), C (10, -3), D (9,-7), and E (8,-8), shown in Figure 5. The manipulator follows a cubic trajectory from one task point to another, given by Equation (12)

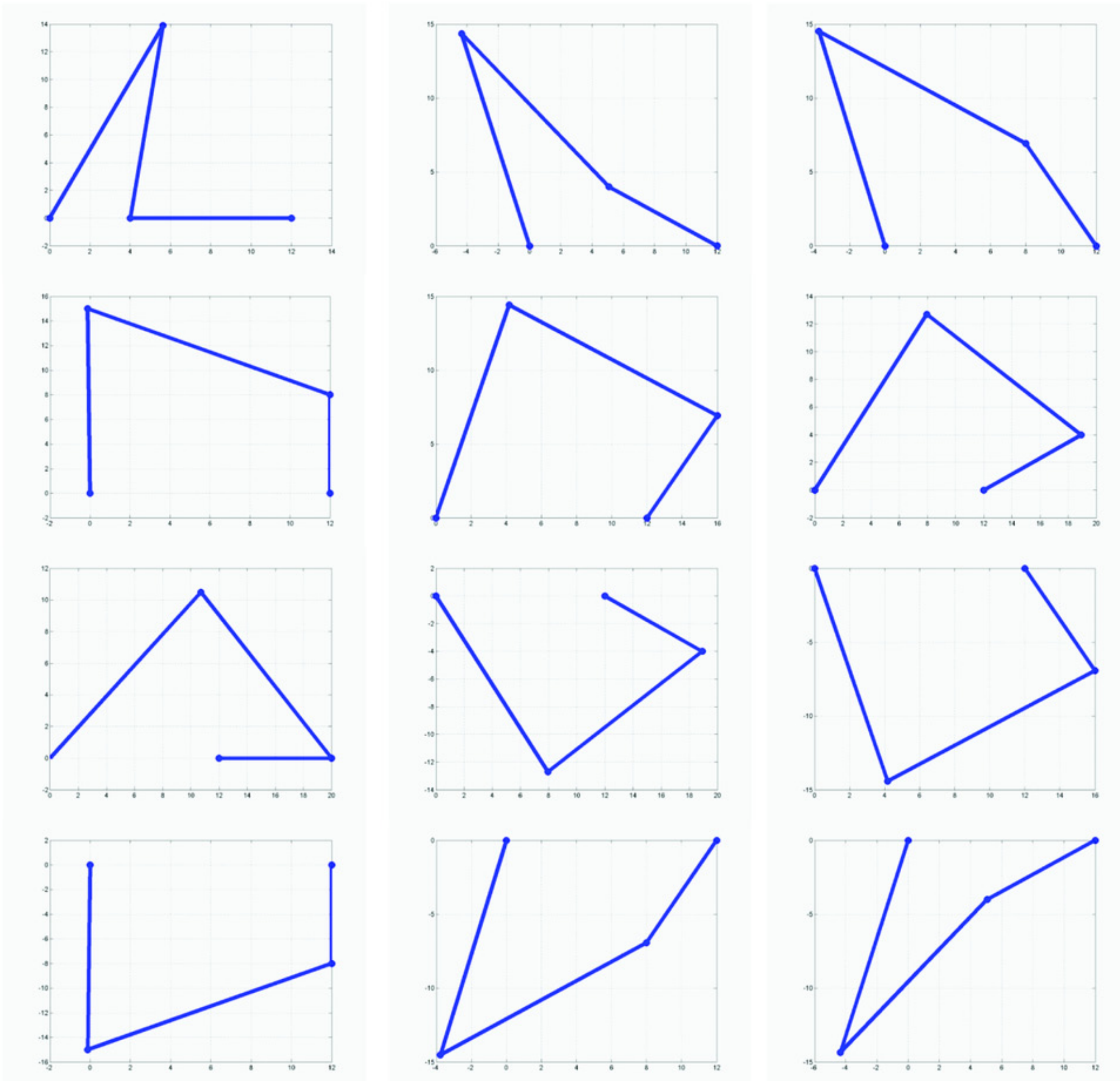
$$y = -0.2292x^3+7.625x^2-78.83x+252 \quad (12)$$

Following the steps in the proposed algorithm, we have:

1. The minimum and maximum distances from the base while following the trajectory are 10 and 20, such that $10 \leq d \leq 20$, assuming the manipulator will be based at the origin O (0,0).
2. We chose $l_3 = 8$ such that $l_3 < d_{min}$.
3. From Equation (12), (13) we have $l_1 = 15$ and $l_2 = 14$.

Using the above generated link lengths, we calculate the Dexterity Index (D) of the manipulator at all points on its workspace. Assuming that the manipulator operates in the XY-plane,

Figure 3. Different orientations of the manipulator about a set task point



the maximum value of the Dexterity index (D) = (1/3) and the maximum value of the $d_z = 1$ while $d_x = d_y = 0$.

Case 1: Ideal Case

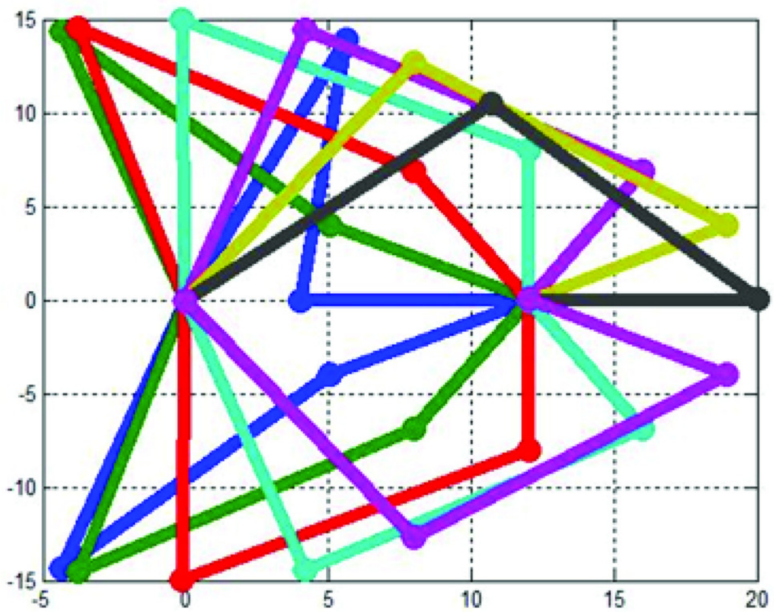
We simulate the manipulator to generate the dexterity plot in the ideal case. We assume that all three joints are completely revolute and the joint angles are not limited. The joint angles are only limited such that the links do not overlap each other, should the angle between the links be close to zero.

In this case, the manipulator has a very symmetric dexterous workspace spread over the four quadrants, as seen in Figure 6. At points in the

Table 1. Grashof's criterion with varying d

Link Lengths	Grashof's Criterion	Shortest Link
$l_1>l_2>l_3>d$	$l_1+d\leq l_2+l_3$	d
$l_1>l_2>d>l_3$	$l_1+l_3\leq d+l_2$	l_3
$l_1>d>l_2>l_3$	$l_1+l_3\leq d+l_2$	l_3
$d>l_1>l_2>l_3$	$d+l_3\leq l_1+l_2$	l_3

Figure 4. Manipulator demonstrating full dexterity about a given task point



workspace where Grashof's criterion is satisfied, the shortest link in the chain is completely revolute, therefore the end-effector can have infinite orientations about the point. As seen in Figures 4 and 5, the manipulator has maximum Dexterity Index (D) = 0.33 or d_z =1, when $10\leq d\leq 20$, which was the design criteria. The variance of dexterity index in the workspace can be seen more clearly in the sectional view, as seen in Figure 7.

The behavior of the manipulator within its dexterous workspace can be better analyzed by dividing the plot into four regions, as shown in Figure 8:

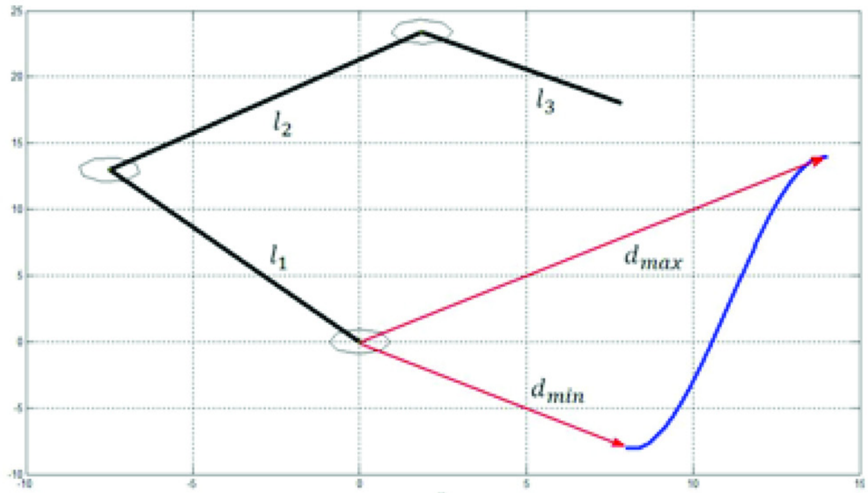
Region 1: $0\leq d\leq l_3$

Within this region, the shortest link in the chain is not l_3 but d. In this case, when Grashof's criterion is satisfied for a point the link, will be fully revolute. However, since is the distance between the end-effector and base, and is not a physical link, it is fixed and non-revolute. The points within this region will have limited inverse kinematic solutions and hence, low manipulability.

Region 2: $l_3<d\leq l_1+l_2-l_3$

In this region, Grashof's criterion is satisfied for all values of. For all points within this region the shortest link in the chain (end-effector) is completely revolute for all values of , thereby

Figure 5. Desired manipulator trajectory



able to have infinite orientations about the point and hence, maximum dexterity.

Region 3: $l_1+l_2-l_3 < d \leq l_1+l_2+l_3$

Here, the value of d is so large that the sum of the largest link (d) and the shortest link (l_3) is no longer less than the sum of the other two links. In this region, the manipulator behaves as a non-Grashof linkage. As $d \rightarrow l_1+l_2+l_3$, the boundaries of the workspace, the dexterity decreases because there are very few inverse kinematic solutions for points close to edges of the workspace.

Region 3: $d > l_1+l_2+l_3$

This region is beyond the limits of the end-effector.

Case 2: Ideal Case; Changed Order

As noted earlier, we know that the dexterity of the manipulator is also a function of the relative positioning of the links. To prove this, in this case we change the order of the links by swapping the lengths of links l_2 and l_3; ($l_2 = 8$ and $l_3 = 14$). The new link order for this case is $l_1 = 15$; $l_2 = 8$ and $l_3 = 14$.

Unlike the previous case, the high dexterity band is very thin, as seen in Figure 9. This is so because, when Grashof's criterion is met, the shortest link in the chain becomes fully revolute,

Figure 6. 3D plot of the dexterity index in the workspace

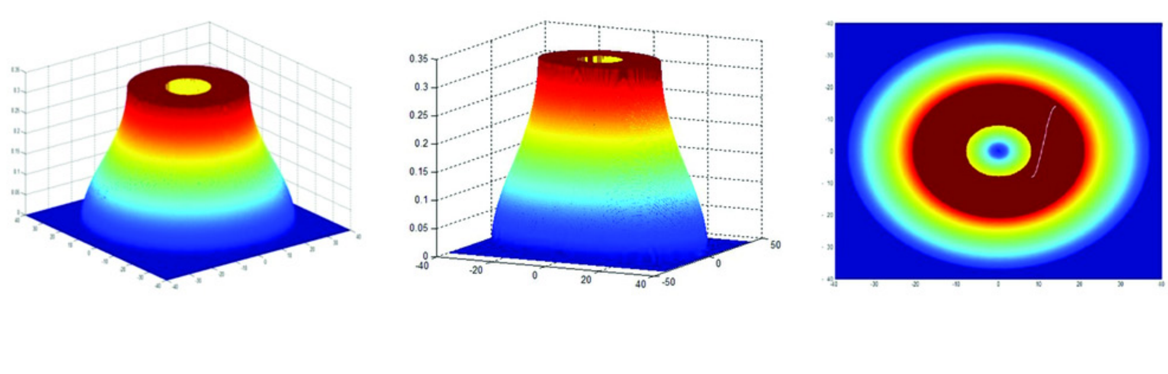


Figure 7. Sectional view of the dexterity index (d)

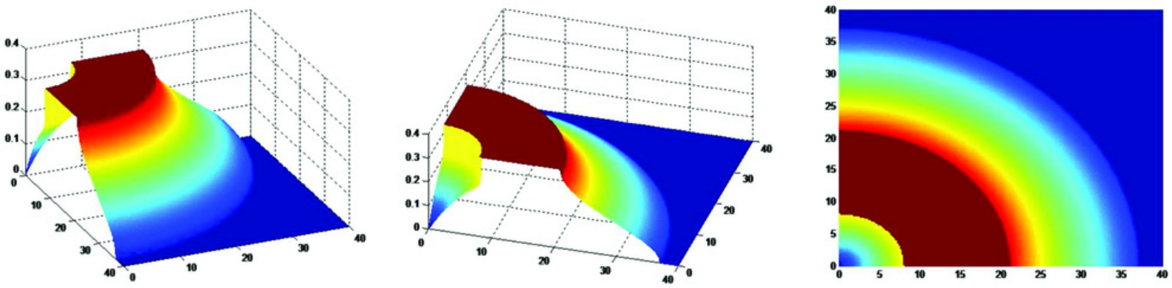
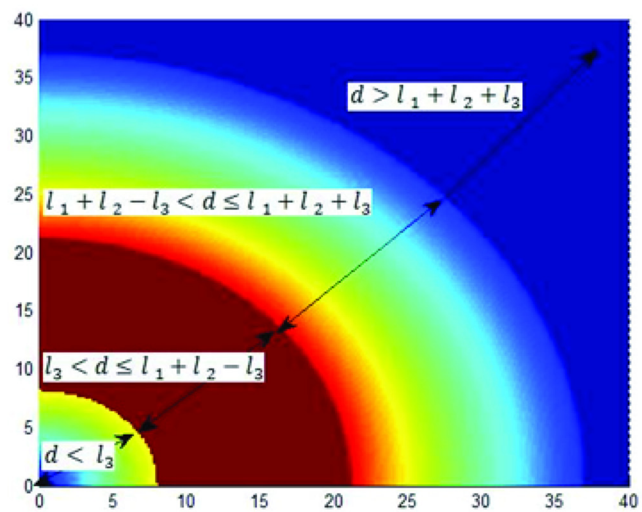


Figure 8. Different regions in the dexterity plot



which in this case is the second link (l_2) and not the last link (l_3). The high dexterity band (Region 2) is given by:

$$l_3 < d \leq l_1 + l_2 - l_3 \tag{13}$$

which in this case, will be $8 < d \leq 9$

Figure 10 gives a comparative view of the dexterity plots in Case 1 and Case 2. As seen in Figure 10, with the positioning of the links changed, the boundaries of the workspace remain the same but the maximum dexterity region is significantly diminished in Case 2. This simulation reinforces the fact that the dexterity index of a manipulator is also a function of the relative positioning of the links and not just the link lengths.

Case 3: Limited Joint Angles

Grashof's criterion assumes that the joints are completely flexible, i.e., able to attain any angle between 0 and 2π. However, this is not possible in practice. The rotation of joints is often limited due to various mechanical and/or workspace constraints. In this case, we consider a practical scenario where the range of the joint angles assumes practical values.

Figure 9. Dexterity plot with the changed order of the links

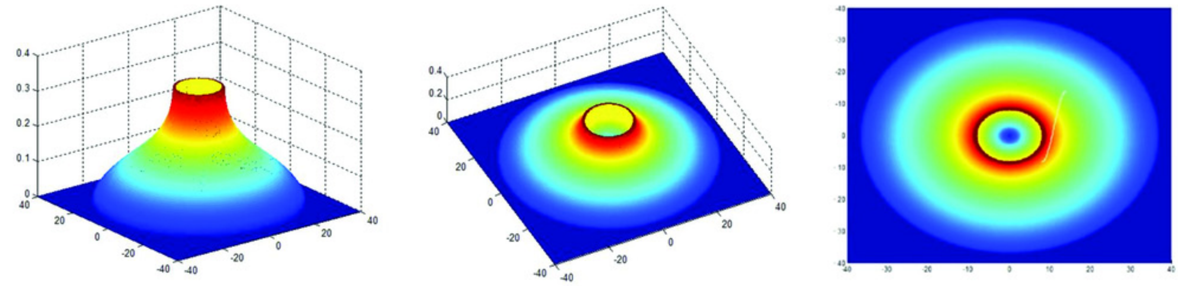
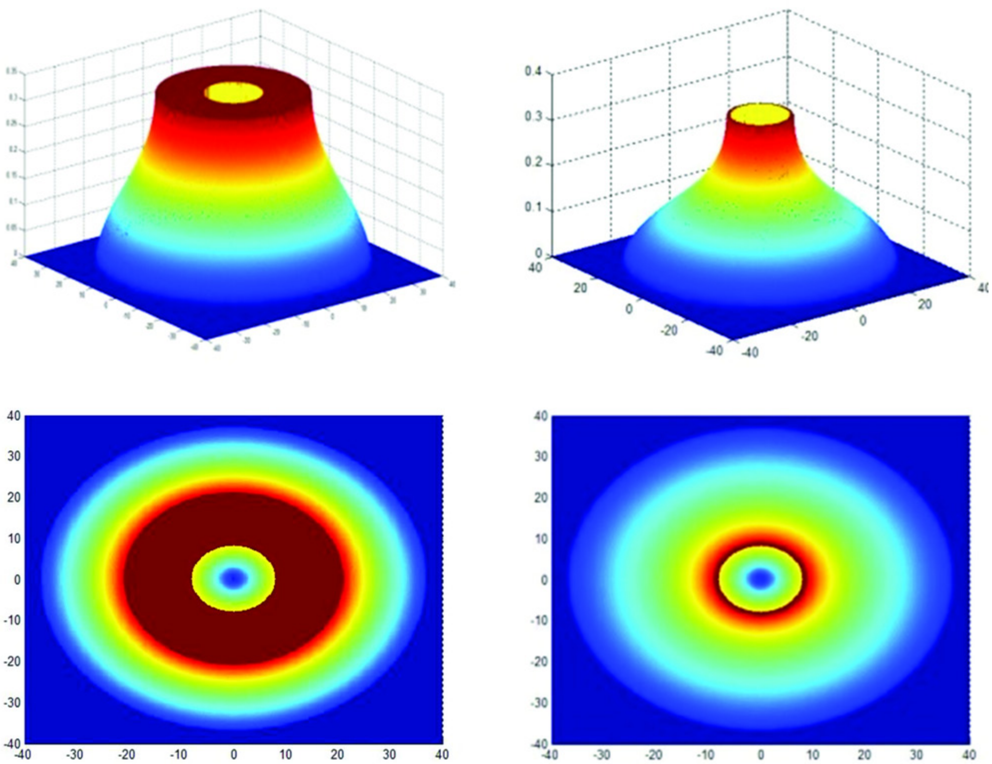


Figure 10. Comparative views of workspace dexterity (case 1 vs. case 2)



$-150 \leq \theta_1 \leq 150$
$-150 \leq \theta_2 \leq 150$ and
$-175 \leq \theta_3 \leq 175$

As seen in Figure 11, the dexterity plot of the left half is distorted. This is due to the limitations of the first two joint angles. With limited joint angles, there will be fewer orientations possible about the point in the workspace when compared to Case 1.

EXAMPLE 2: INDEPENDENT TASK POINTS

Let us consider another design example where we need to design an optimal manipulator configuration to have maximum dexterity while following the independent task points A(16,19), B(18,0), C(-8,-25), D(-15,11), E(-10,16), F(20,-21), and G(-7,27). We assume that the manipulator is at the origin of coordinate frame O (0, 0). Following the steps in the proposed algorithm, we have:

1. The minimum and maximum distances of the points from the manipulator base are 29 (Point F) and 18 (Point B), such that $18 \leq d \leq 29$
2. We chose l_3 = 15 such that $l_3 < d_{min}$
3. From Equation (12), (13) we have l_1 = 23 and l_2 =22

The simulation plots for the manipulator under all three cases can be seen in Figure 12. In Case 1 where ideal joints are assumed, we see that all the task points are within the most dexterous band. The design ensures that the manipulator will be able to attain every possible orientation about each of the task points.

In Case 2 the lengths l_2 and l_3 are interchanged. As expected, the most dexterous band is greatly reduced and is very thin in this case. All the task points lie outside the most dexterous band. This simulation once again proves that in order to achieve maximum dexterous area, the shortest link in the manipulator chain should be the end-effector or the last link.

In Case 3 we assume that the joints are with constraint, i.e. the joints can only attain a range of joint angles. As seen in Figure 12 the dexterity regions are very unsymmetrical when compared to the previous two plots, but the high dexterity bands

Figure 11. Workspace dexterity plot under joint angle constraints

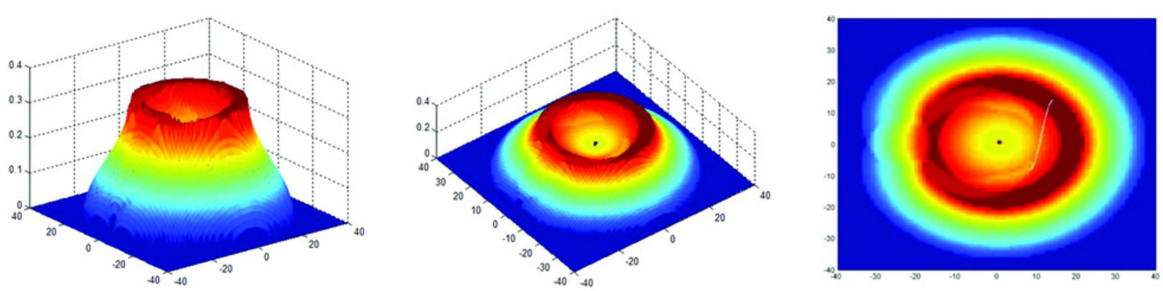
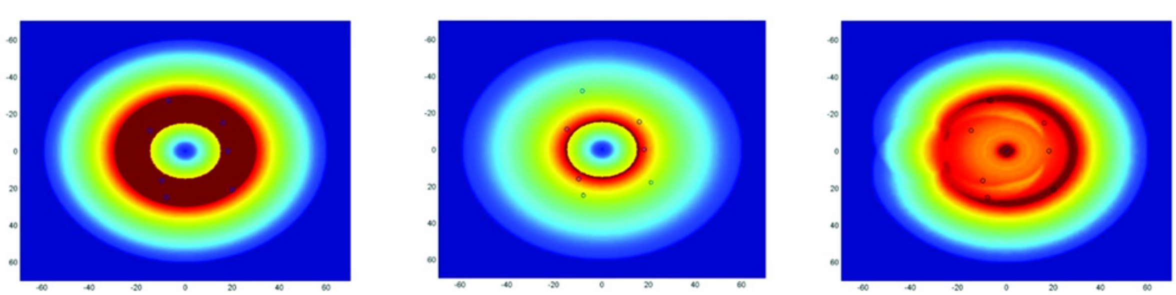


Figure 12. Simulation plots for case 1, 2, & 3



are similar to the bands in Case 1. The dexterity bands are distorted to the left because the point in this region will have limited solutions due the joint angle constraints put in place. In spite of the joint constraint, three task points lie in the most dexterous region while the rest are located in high dexterity band, as in Figure 12.

6. CONCLUSION

In this chapter, we have proposed a simple algorithm for generating optimal link lengths for a three-link planar manipulator. By adding a virtual stationary link in the chain, the three-link manipulator is converted into a four-link closed chain. Using Grashof's criterion, we optimize the link lengths to achieve maximum dexterity in the desired regions of the workspace. Dexterity index plots of the workspace generated by simulating the optimal manipulator configurations meet our design. Simulations with the link order changed (Case 2) have shown that the manipulator has a maximum high dexterity area when the shortest link is also the last link (end-effector) of the manipulator. The dexterity index plots of the workspace generated by applying Grashof's criterion help in better task placement and trajectory planning, especially when the manipulators mobility is limited due to joint angle limitations (Case 3).

7. FUTURE RESEARCH

In our future research, we would focus on applying the extended Grashof's criterion for N-bar chains for the prototyping of minimum degree of freedom (DoF) manipulators with optimal link lengths for accomplishing a specified task.

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